Theory of Charged Vector Mesons Interacting with the Electromagnetic Field

T. D. LEE AND C. N. YANG
Institute for Advanced Study, Princeton, New Jersey
(Received May 29, 1962)

It is shown that starting from the usual canonical formalism for the electromagnetic interaction of a charged vector meson with arbitrary magnetic moment one is led to a set of rules for Feynman diagrams, which appears to contain terms that are both infinite and noncovariant. These difficulties, however, can be circumvented by introducing a ξ-limiting process which depends on a dimensionless positive parameter $\xi \to 0$. Furthermore, by using the mathematical artifice of a negative metric the theory becomes renormalizable (for $\xi > 0$).

1. INTRODUCTION

THE problem of a charged vector meson interacting with the electromagnetic field and other fermion fields has been discussed rather extensively in the past.[1,2] However, in the literature, there does not seem to exist any systematic study of the general case in which the charged vector meson could have an arbitrary magnetic moment. Furthermore, the question of the renormalizability of a theory of charged vector meson has not been studied in detail. The recent speculations in weak interactions[3] and the possibility that, perhaps, a vector meson $W^{\pm}$ could be produced by high-energy neutrinos[4] through its electromagnetic and weak interactions give new interest to these problems.

In this paper, an attempt is made to study these problems. We begin with a discussion of the derivation of Feynman rules for the general case of the interactions between the electromagnetic field and charged vector mesons with arbitrary magnetic moment. It turns out that by starting from the conventional canonical formalism[1] and using the Dyson-Wick procedure,[5] one obtains a set of rules for the Feynman graphs which contains terms that appear to be both infinite and noncovariant. It is then shown that this formal difficulty can be resolved by introducing a limiting process (called ξ-limiting process) which depends on a positive parameter $\xi \to 0$. The resulting rules for Feynman graphs in the ξ-limiting process become completely covariant. However, the theory continues to be divergent in a nonrenormalizable way. To remedy this, the artifice of a negative metric is introduced which makes the parameter ξ take on the role of a regulator. The final theory for $\xi > 0$ is both covariant and renormalizable. It is further shown that while the introduction of a negative metric destroys unitarity, the S matrix remains unitary as long as the total energy of the system is less than $\xi^{-\frac{1}{2}}$ times the mass of the meson.

The derivations of Feynman rules are sometimes rather complicated, because of the presence of time derivatives of field variables in the interaction Lagrangian. These detailed derivations are all given in the Appendices. As an illustration, the derivation of Feynman rules for the simple and well-known case of a charged vector meson field interacting with Fermion fields is included in Appendix A.

Strictly speaking, because of divergences there does not exist a "true" charged vector meson theory. Any theory of the charged vector meson is in this sense a separate proposal not derivable from a "true" theory. What gives the confidence that the renormalization procedure of the photon-electron interaction enjoys is, besides the impressive and accurate experimental verifications, the belief that any covariant proposal would lead essentially to the results of the usual renormalization procedure. For the charged vector meson, it is our present belief that, with the ξ-limiting process and the indefinite metric, one has a covariant theory that in some measure gives that part of the properties of the charged vector meson which is independent of specific details at very small distances.

2. CANONICAL FORMALISM

2.1 Lagrangian

We discuss a charged vector meson field φ_μ in interaction with the electromagnetic field A_μ. The charged vector mesons is assumed to possess an arbitrary magnetic moment, and is called $W^{\pm}$. The Lagrangian density of the system is[6] ($\hbar = c = 1$)

$$\mathcal{L} = -\frac{1}{2}\left(\frac{\partial A_\mu}{\partial x_\nu}\right)\left(\frac{\partial A_\mu}{\partial x_\nu}\right) - \tfrac{1}{2} G_{\mu\nu}{}^* G_{\mu\nu} - m^2 \varphi_\mu{}^* \varphi_\mu - ie\kappa F_{\mu\nu} \varphi_\mu{}^* \varphi_\nu, \quad (1)$$

[1] See, for example, G. Wentzel, *Quantum Theory of Fields* (Interscience Publishers, Inc., New York, 1949). In this paper we start with the formulation of vector meson field given in Wentzel's book.

[2] Feynman rules for charged vector mesons have been given by R. P. Feynman, Phys. Rev. **76**, 769 (1949) using his method of space-time approach of field theory. More detailed discussions on Feynman rules for charged vector mesons in the β formalism were given by C. N. Yang and G. Feldman, Phys. Rev. **79**, 972 (1950). See also T. Kinoshita and Y. Nambu, Progr. Theoret. Phys. **5**, 473, 749 (1950); P. T. Matthews, Phys. Rev. **76**, 1657 (1949).

[3] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

[4] T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960). See also B. Pontecorvo and R. M. Ryndin, Dubna Report D-577 (unpublished).

[5] F. J. Dyson, Phys. Rev. **75**, 486 (1949); G. C. Wick, *ibid.* **80**, 268 (1950).

[6] Throughout this paper we use the following notations: All boldface letters such as **k**, **r**, **A**, $\boldsymbol{\varphi}$, etc., denote three-vectors. The fourth component ik_0 of the four-momentum k_μ is pure imaginary. All Greek subscripts μ, ν, $\cdots$ vary from 1 to 4 and all Roman subscripts i, j, $\cdots$ vary from 1 to 3. Repeated indices are to be summed over.

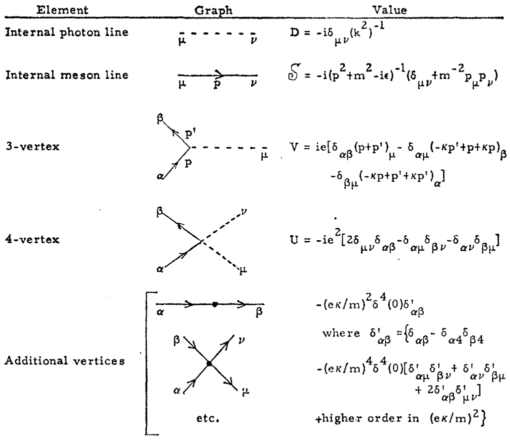


FIG. 1. Feynman diagram in momentum representation for Lagrangian (1). Each closed internal loop consisting of meson and photon lines gives rise to one integration $(2\pi)^{-4}\int[\cdots]dk_0d^3k$. A diagram includes specific assignments of momenta and polarization to all external lines, but not internal lines. The weight of each different diagram is s^{-1} where s is the symmetry number defined as follows: Label each internal line with a different integer: 1, 2, $\cdots N$. There are $N!$ different ways of labeling. Some of these labelings may lead to labeled diagrams with identical topological structure. s is simply the number of such different labelings that lead to the same labeled diagram.

where $*$ = Hermitian conjugate times $(-1)^n$, n = number of "4" subscripts,

$$\begin{aligned} F_{\mu\nu} &= (\partial/\partial x_\mu)A_\nu - (\partial/\partial x_\nu)A_\mu, \\ G_{\mu\nu} &= \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu, \\ G_{\mu\nu}^* &= \partial_\mu^* \varphi_\nu^* - \partial_\nu^* \varphi_\mu^*, \\ \partial_\mu &= \partial/\partial x_\mu - ieA_\mu, \\ \partial_\mu^* &= \partial/\partial x_\mu + ieA_\mu, \end{aligned} \tag{2}$$

and κ is a constant. The magnetic moment $\mathfrak{M}$ and the quadrupole moment Q of W^+ is given by

$$\mathfrak{M} = (1+\kappa)(e/2m)\mathbf{S} \tag{3}$$

and

$$Q \equiv \int (3z^2 - r^2)\rho d^3r = -(e\kappa/m^2), \tag{4}$$

where $\mathbf{S}$ is the spin of W and ρ is the static charge density for the state $S_z = +1$.

The equation of motion for $W^\pm$ is

$$\partial_\mu G_{\mu\nu} - m^2\varphi_\nu + ie\kappa\varphi_\mu F_{\mu\nu} = 0. \tag{5}$$

2.2 Feynman Diagram

In Appendix B we carry out in detail the canonical formalism starting from the Lagrangian above: The fields[6] $\boldsymbol{\varphi}$, $\boldsymbol{\varphi}^*$, $\mathbf{A}$, and A_4 will be treated as independent canonical "coordinates." φ_4 and φ_4^* will be treated as dependent coordinates with the aid of (5). One then obtains a Hamiltonian for the system. By a unitary transformation one goes over into the interaction representation. Feynman diagrams will then be obtained through the Dyson-Wick[5] procedure.

The result of these considerations is as follows. A Feynman diagram in the present case is very much like that for the electron-photon interaction, except that there are now three kinds of vertices. The values of these vertices and the propagators, in momentum representation, are listed in Fig. 1 (proved in Appendix C).

For the purpose of easy memory we remark that the three-vertex and four-vertex functions are the matrix elements of

$$\begin{aligned} -i[\mathfrak{L}(e=0) - \mathfrak{L}(e)] &= e\kappa F_{\nu\mu}\varphi_\nu^*\varphi_\mu + \tfrac{1}{2}e\bigg[(A_\nu\varphi_\mu^* - A_\mu\varphi_\nu^*) \\ &\quad \times\left(\frac{\partial}{\partial x_\nu}\varphi_\mu - \frac{\partial}{\partial x_\mu}\varphi_\nu\right) - \text{Herm. conj.}\bigg] \\ &\quad - \tfrac{1}{2}ie^2(A_\nu\varphi_\mu - A_\mu\varphi_\nu)(A_\nu\varphi_\mu^* - A_\mu\varphi_\nu^*), \end{aligned} \tag{6}$$

where all operators are regarded as free fields. However, this very simple rule does not give the whole story, as the presence of the additional vertices in Fig. 1 explicitly shows.

The additional vertices are all divergent and are explicitly noncovariant. For a given process, to the lowest order in e the Feynman diagram does not contain closed loops, nor does it contain any of the additional vertices. For a higher order diagram, because of the divergent nature of the integral, the integration dk_0 gives, in addition to the usual pole contributions, contributions due to the closing of the integration contour at ∞ in the complex k_0 plane. As is discussed in Appendix E, the divergent and noncovariant vertices of Fig. 1 are the results of such extra integration contributions at infinity. Moreover, they are present only if $\kappa \neq 0$. [This is because in the usual canonical formalism the components of $\boldsymbol{\varphi}$, $\boldsymbol{\varphi}^*$ are treated as coordinates, but φ_4 and φ_4^* are regarded as functions of $\boldsymbol{\varphi}$, $\boldsymbol{\varphi}^*$ and their conjugate momenta. Therefore, the interaction term $-ie\kappa F_{\mu\nu}\varphi_\mu^*\varphi_\nu$ in the Lagrangian (1) appears to contain more than one time derivative of the field variables which gives rise to these additional vertices. In this paper, Lagrangians which contain terms with more than two time derivatives of the field variables are not considered.]

3. ξ-LIMITING FORMALISM

The origin of these complications, therefore, lies in the fact that φ_4 is not treated on equal footing as the components of $\boldsymbol{\varphi}$. To circumvent this difficulty we add a term to the Lagrangian proportional to a dimensionless parameter ξ and then take the limit $\xi \to 0$.

3.1 Lagrangian

Instead of (1) we thus have a Lagrangian density with an additional term:

$$\mathcal{L}_\xi = -\xi(\partial_\mu{}^*\varphi_\mu{}^*)(\partial_\nu\varphi_\nu) - \frac{1}{2}\left(\frac{\partial A_\mu}{\partial x_\nu}\right)\left(\frac{\partial A_\mu}{\partial x_\nu}\right) - \tfrac{1}{2}G_{\mu\nu}{}^*G_{\mu\nu} - m^2\varphi_\mu{}^*\varphi_\mu - ie\kappa F_{\mu\nu}\varphi_\mu{}^*\varphi_\nu. \quad (7)$$

The equation of motion for $W^\pm$ becomes

$$\partial_\mu G_{\mu\nu} - m^2\varphi_\nu + \xi\partial_\nu(\partial_\mu\varphi_\mu) + ie\kappa\varphi_\mu F_{\mu\nu} = 0.$$

The Lagrangian density (7) can now be treated by the canonical formalism in a straightforward way. We state the result as follows.

3.2 Free Meson Field

By using (7) and setting $e=0$ one obtains the following free Hamiltonian H_0:

$$H_0 = \boldsymbol{\pi}\cdot\boldsymbol{\pi}^* + \xi^{-1}\pi_4\pi_4{}^* + m^2\varphi_\mu{}^*\varphi_\mu + (\nabla\times\boldsymbol{\varphi})\cdot(\nabla\times\boldsymbol{\varphi}^*) + i(\boldsymbol{\pi}\cdot\nabla\varphi_4 + \boldsymbol{\pi}^*\cdot\nabla\varphi_4{}^* - \pi_4\nabla\cdot\boldsymbol{\varphi} - \pi_4{}^*\nabla\cdot\boldsymbol{\varphi}^*), \quad (8)$$

where π_μ and $\pi_\mu{}^*$ are, respectively, the conjugate momenta of φ_μ and $\varphi_\mu{}^*$. The commutation relations at equal time are given by

$$[\pi_\mu(\mathbf{r},t), \varphi_\nu(\mathbf{r}',t)] = -i\delta_{\mu\nu}\delta^3(\mathbf{r}-\mathbf{r}'),$$
$$[\pi_\mu{}^*(\mathbf{r},t), \varphi_\nu{}^*(\mathbf{r}',t)] = -i\delta_{\mu\nu}\delta^3(\mathbf{r}-\mathbf{r}'), \quad (9)$$

and all other commutators between φ_μ, $\varphi_\mu{}^*$, π_μ, $\pi_\mu{}^*$ are zero. The free field corresponds to a system of uncoupled mesons which can be described by the annihilation operators

$a_{\mathbf{k}}{}^t$, $b_{\mathbf{k}}{}^t$ for + and − *transverse* (spin 1) mesons, $t=1, 2$
$a_{\mathbf{k}}{}^l$, $b_{\mathbf{k}}{}^l$ for + and − *longitudinal* (spin 1) mesons,
$a_{\mathbf{k}}{}^s$, $b_{\mathbf{k}}{}^s$ for + and − scalar (spin 0) mesons,

and their Hermitian conjugates, the creation operators, $a_{\mathbf{k}}{}^{s\dagger}$, $b_{\mathbf{k}}{}^{s\dagger}$, $a_{\mathbf{k}}{}^{l\dagger}$, etc. In terms of these operators one has the following representation:

$$\boldsymbol{\varphi} = \sum_{\mathbf{k},t}(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^t\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t) + b_{-\mathbf{k}}{}^{t\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)]\mathbf{e}_{\mathbf{k}}{}^t + \sum_{\mathbf{k}}(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^l\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t) + b_{-\mathbf{k}}{}^{l\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)](\omega\hat{k}/m) - \sum_{\mathbf{k}}(2\Omega\nu)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^s\exp(i\mathbf{k}\cdot\mathbf{r}+i\nu t) + b_{-\mathbf{k}}{}^{s\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}-i\nu t)](\mathbf{k}/m),$$

$$\varphi_4 = \sum_{\mathbf{k}}(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^l\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t) - b_{-\mathbf{k}}{}^{l\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)](i|\mathbf{k}|/m) + \sum_{\mathbf{k}}(2\Omega\nu)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^s\exp(i\mathbf{k}\cdot\mathbf{r}+i\nu t) - b_{-\mathbf{k}}{}^{s\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}-i\nu t)](i\nu/m), \quad (10)$$

$$\boldsymbol{\pi} = \sum_{\mathbf{k},t} i(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^{t\dagger}\exp(-i\mathbf{k}\cdot\mathbf{r}+i\omega t) - b_{-\mathbf{k}}{}^t\exp(-i\mathbf{k}\cdot\mathbf{r}-i\omega t)]\omega\mathbf{e}_{\mathbf{k}}{}^t + \sum_{\mathbf{k}} i(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^{l\dagger}\exp(-i\mathbf{k}\cdot\mathbf{r}+i\omega t) - b_{-\mathbf{k}}{}^l\exp(-i\mathbf{k}\cdot\mathbf{r}-i\omega t)](m\hat{k}),$$

$$\pi_4 = \sum_{\mathbf{k}}(2\Omega\nu)^{-\frac{1}{2}}m[a_{\mathbf{k}}{}^{s\dagger}\exp(-i\mathbf{k}\cdot\mathbf{r}-i\nu t) + b_{-\mathbf{k}}{}^s\exp(-i\mathbf{k}\cdot\mathbf{r}+i\nu t)],$$

and $\varphi_\mu{}^*$, $\pi_\mu{}^*$ are related to the Hermitian conjugates $\varphi_\mu{}^\dagger$, $\pi_\mu{}^\dagger$ of φ_μ, π_μ by

$$\boldsymbol{\varphi}^* = \boldsymbol{\varphi}^\dagger, \qquad \boldsymbol{\pi} = \boldsymbol{\pi}^\dagger,$$
$$\varphi_4{}^* = -\varphi_4{}^\dagger, \quad \pi_4{}^* = -\pi_4{}^\dagger. \quad (11)$$

In these formulas, $e_{\mathbf{k}}{}^1$, $e_{\mathbf{k}}{}^2$, and $\hat{k} = |\mathbf{k}|^{-1}\mathbf{k}$ form a right-handed orthonormal set of unit vectors,

$$\omega = (\mathbf{k}^2+m^2)^{\frac{1}{2}} > 0, \quad \nu = (\mathbf{k}^2+\xi^{-1}m^2)^{\frac{1}{2}} > 0, \quad (12)$$

and Ω is the normalization volume. In terms of these annihilation and creation operators H_0 becomes

$$H_0 = \sum_{\mathbf{k},t}\omega(a_{\mathbf{k}}{}^{t\dagger}a_{\mathbf{k}}{}^t + \tfrac{1}{2}) + \sum_{\mathbf{k}}\omega(a_{\mathbf{k}}{}^{l\dagger}a_{\mathbf{k}}{}^l + \tfrac{1}{2}) - \sum_{\mathbf{k}}\nu(a_{\mathbf{k}}{}^{s\dagger}a_{\mathbf{k}}{}^s + \tfrac{1}{2}) + \text{same terms with } a\to b. \quad (13)$$

These formulas show that the additional ξ-dependent term in the Lagrangian introduces scalar mesons with a negative energy

$$-\nu = -(\mathbf{k}^2+\xi^{-1}m^2)^{\frac{1}{2}},$$

which approaches $-\infty$ as $\xi\to 0$.

3.3 Hamiltonian in Interaction Representation

The indefiniteness of the Hamiltonian makes it very doubtful that after the introduction of the coupling e when different meson states are coupled, the theory can still make physical sense. We try to remedy this by introducing a negative metric in Sec. 4. For clarity of presentation, we ignore this difficulty for the time being and proceed with the canonical formalism. All the four components of φ_μ are now regarded as canonical coordinates. In the interaction representation, the space-time dependences of the operators φ_μ and A_μ are the same as that of the free ones. In terms of these operators the interaction Hamiltonian becomes[6]

$$H_{\text{int}}=-ieA_\mu[g_{\mu\nu}^*\varphi_\nu-g_{\mu\nu}\varphi_\nu^*]-ie\xi A_\mu\left[\varphi_\mu\left(\frac{\partial}{\partial x_\nu}\varphi_\nu^*\right)-\varphi_\mu^*\left(\frac{\partial}{\partial x_\nu}\varphi_\nu\right)\right]+e^2[(A_jA_j)(\varphi_k^*\varphi_k)-(A_j\varphi_j)(A_k\varphi_k^*)]$$
$$+ie\kappa F_{\mu\nu}\varphi_\mu^*\varphi_\nu+\tfrac{1}{2}e^2\kappa^2[(\varphi_4^*)^2\varphi_j\varphi_j+(\varphi_4)^2\varphi_j^*\varphi_j^*-2(\varphi_4^*\varphi_4)(\varphi_j^*\varphi_j)],\quad(14)$$

where

$$g_{\mu\nu}=\left(\frac{\partial}{\partial x_\mu}\varphi_\nu\right)-\left(\frac{\partial}{\partial x_\nu}\varphi_\mu\right)\qquad(15)$$

and

$$g_{\mu\nu}^*=\left(\frac{\partial}{\partial x_\mu}\varphi_\nu^*\right)-\left(\frac{\partial}{\partial x_\nu}\varphi_\mu^*\right).$$

3.4 Feynman Diagram

Using the Dyson-Wick[5] procedure one obtains the Feynman diagrams for the Lagrangian (7). The values of the propagators and vertices are listed in Fig. 2 (proved in Appendix D).

3.5 Divergenceless Current Density

The Lagrangian (7) is[7] gauge invariant. Therefore the current density is divergenceless. An explicit proof of this fact can be obtained from an examination of the vertices and propagators of Fig. 2, in the same spirit as the corresponding proof[8] for the electromagnetic field-electron interaction. In the present case, the proof is slightly more complicated because of the momentum dependence of the vertices V which generates terms canceled by the vertices U.

4. NEGATIVE METRIC

In the ξ formalism, the propagator $\mathcal{S}$ in Fig. 2 consists of two parts: a spin-one part $-i(p^2+m^2-i\epsilon)^{-1}\times(\delta_{\mu\nu}+m^{-2}p_\mu p_\nu)$ and a spin-zero part $i(p^2+\xi^{-1}m^2+i\epsilon)^{-1}\times(m^{-2}p_\mu p_\nu)$. At first sight, it might appear that the presence of the spin-zero part acts like a regulator; therefore, we might have a renormalizable theory for $\xi\neq0$. That this is not the case can easily be seen by noticing the different signs $\pm i\epsilon$ in these two parts of the propagator. More explicitly, $\mathcal{S}$ can be written as

$$\mathcal{S}=S-2\pi i(m^{-2}p_\mu p_\nu)\delta(p^2+\xi^{-1}m^2),\qquad(16)$$

where

$$S=-i(p^2+m^2-i\epsilon)^{-1}(\delta_{\mu\nu}+m^{-2}p_\mu p_\nu)+i(p^2+\xi^{-1}m^2-i\epsilon)^{-1}(m^{-2}p_\mu p_\nu).\qquad(17)$$

The second term on the right-hand side of (16) makes the theory discussed in the above section divergent in an unrenormalizable way. In the ξ-limiting formalism, in order to give a meaningful discussion of the limit $\xi\to0$, finite physical results must be first obtained before taking the limit. To achieve this we introduce a negative metric in the Hilbert space.

4.1 ξ-Limiting Formalism with Negative Metric

We start with the identical Lagrangian given by (7), except that φ_μ^* and $G_{\mu\nu}^*$ are replaced by

$$\varphi_\mu^\star\equiv\eta^{-1}\varphi_\mu^*\eta,\qquad G_{\mu\nu}^\star\equiv\eta^{-1}G_{\mu\nu}^*\eta,\qquad(18)$$

respectively. Following the notation of Pauli,[9] we use η to represent the metric of the Hilbert space. It becomes clear that in order to change the sign of $(i\epsilon)$ in the spin-zero part of the propagator $\mathcal{S}$ the metric η in the *interaction representation* is chosen to be

$$\eta=(-1)^{N_s},\qquad(19)$$

where N_s is the total number of scalar mesons.

For clarity, we discuss first the free-field case ($e=0$) and then the general case in the interaction representation.

4.2 Free Meson Fields

Identical with (8) and (9) except for the replacement (18), the free Hamiltonian H_0 for the present case is

$$H_0=\boldsymbol{\pi}\cdot\boldsymbol{\pi}^\star+\xi^{-1}\pi_4\pi_4^\star+m^2\varphi_\mu\varphi_\mu^\star+(\nabla\times\boldsymbol{\varphi})\cdot(\nabla\times\boldsymbol{\varphi}^\star)+i(\boldsymbol{\pi}\cdot\nabla\varphi_4+\boldsymbol{\pi}^\star\cdot\nabla\varphi_4^\star-\pi_4\nabla\cdot\boldsymbol{\varphi}-\pi_4^\star\nabla\cdot\boldsymbol{\varphi}^\star),\quad(20)$$

and the commutation relations are

$$[\pi_\mu(\mathbf{r},t),\varphi_\nu(\mathbf{r}',t)]=-i\delta_{\mu\nu}\delta^3(\mathbf{r}-\mathbf{r}'),\qquad [\pi_\mu^\star(\mathbf{r},t),\varphi_\nu^\star(\mathbf{r}',t)]=-i\delta_{\mu\nu}\delta^3(\mathbf{r}-\mathbf{r}').\qquad(21)$$

All other equal-time commutators between φ_μ, $\varphi_\mu^\star$, π_μ, $\pi_\mu^\star$ are zero. We list the explicit representation of these

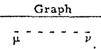
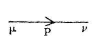
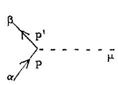
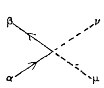
Element	Graph	Value
Internal photon line		$D=-i\delta_{\mu\nu}(k^2)^{-1}$
Internal meson line		$\mathcal{S}=-i(p^2+m^2-i\epsilon)^{-1}(\delta_{\mu\nu}+m^{-2}p_\mu p_\nu)+i(p^2+\xi^{-1}m^2+i\epsilon)^{-1}(m^{-2}p_\mu p_\nu)$
3-vertex		$V=ie[\delta_{\alpha\beta}(p+p')_\mu-\delta_{\alpha\mu}(-\kappa p'+p+\kappa p-\xi p')_\beta-\delta_{\beta\mu}(-\kappa+p'+\kappa p'-\xi p)_\alpha]$
4-vertex		$U=-ie^2[2\delta_{\mu\nu}\delta_{\alpha\beta}-(1-\xi)\delta_{\alpha\mu}\delta_{\beta\nu}-(1-\xi)\delta_{\alpha\nu}\delta_{\beta\mu}]$

FIG. 2. Feynman diagram in momentum representation for the Lagrangian (7) (see also caption of Fig. 1).

[7] Dr. T. T. Wu first pointed out the advantage of using a gauge-invariant ξ formalism.

[8] R. P. Feynman, Phys. Rev. **76**, 769 (1949).

[9] W. Pauli, Revs. Modern Phys. **15**, 175 (1945).

operators in terms of the annihilation operators $a_{\mathbf{k}}{}^r$, $b_{\mathbf{k}}{}^r$ and their Hermitian conjugates, the creation operators, $a_{\mathbf{k}}{}^{r\dagger}$ and $b_{\mathbf{k}}{}^{r\dagger}$ (where $r=t$, l, s represent, respectively, the uncoupled transverse, longitudinal, and scalar mesons):

$$\begin{aligned}\boldsymbol{\varphi}=&\sum_{\mathbf{k},t}(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^t\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)+b_{-\mathbf{k}}{}^{t\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)]\mathbf{e}_{\mathbf{k}}{}^t\\&+\sum_{\mathbf{k}}(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^l\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)+b_{\mathbf{k}}{}^{l\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)](\omega\hat{k}/m)\\&+\sum_{\mathbf{k}}(2\Omega\nu)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^s\exp(i\mathbf{k}\cdot\mathbf{r}-i\nu t)+b_{-\mathbf{k}}{}^{s\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\nu t)](\mathbf{k}/m),\\ \varphi_4=&\sum_{\mathbf{k}}(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^l\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)-b_{-\mathbf{k}}{}^{l\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)](i|\mathbf{k}|/m)\\&+\sum_{\mathbf{k}}(2\Omega\nu)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^s\exp(i\mathbf{k}\cdot\mathbf{r}-i\nu t)-b_{-\mathbf{k}}{}^{s\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\nu t)](i\nu/m),\qquad(22)\\ \boldsymbol{\pi}=&\sum_{\mathbf{k},t}i(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^{t\dagger}\exp(-i\mathbf{k}\cdot\mathbf{r}+i\omega t)-b_{-\mathbf{k}}{}^t\exp(-i\mathbf{k}\cdot\mathbf{r}-i\omega t)]\omega\mathbf{e}_{\mathbf{k}}{}^t\\&+\sum_{\mathbf{k}}i(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^{l\dagger}\exp(-i\mathbf{k}\cdot\mathbf{r}+i\omega t)-b_{-\mathbf{k}}{}^l\exp(-i\mathbf{k}\cdot\mathbf{r}-i\omega t)]m\hat{k},\\ \pi_4=&\sum_{\mathbf{k}}(2\Omega\nu)^{-\frac{1}{2}}m[a_{\mathbf{k}}{}^{s\dagger}\exp(-i\mathbf{k}\cdot\mathbf{r}+i\nu t)+b_{-\mathbf{k}}{}^s\exp(-i\mathbf{k}\cdot\mathbf{r}-i\nu t)].\end{aligned}$$

$\varphi_\mu{}^\star$ and $\pi_\mu{}^\star$ are related to the Hermitian conjugates $\varphi_\mu{}^\dagger$, $\pi_\mu{}^\dagger$, of φ_μ, π_μ by

$$\begin{aligned}\boldsymbol{\varphi}^\star&=\eta^{-1}\boldsymbol{\varphi}^\dagger\eta, & \boldsymbol{\pi}^\star&=\eta^{-1}\boldsymbol{\pi}^\dagger\eta,\\ \varphi_4{}^\star&=-\eta^{-1}\varphi_4{}^\dagger\eta, & \pi_4{}^\star&=-\eta^{-1}\pi_4{}^\dagger\eta,\end{aligned}\qquad(23)$$

where

$$\eta=\exp[\textstyle\sum_{\mathbf{k}}i\pi(a_{\mathbf{k}}{}^{s\dagger}a_{\mathbf{k}}{}^s+b_{\mathbf{k}}{}^{s\dagger}b_{\mathbf{k}}{}^s)],\qquad(24)$$

and ω, ν, $\mathbf{e}_{\mathbf{k}}{}^t$, $\hat{k}$ are given by (12).

Upon substituting (22) and (23) to (20), the Hamiltonian H_0 becomes

$$\begin{aligned}H_0=&\sum_{\mathbf{k},t}\omega(a_{\mathbf{k}}{}^{t\dagger}a_{\mathbf{k}}{}^t+\tfrac{1}{2})+\sum_{\mathbf{k}}\omega(a_{\mathbf{k}}{}^{l\dagger}a_{\mathbf{k}}{}^l+\tfrac{1}{2})\\&+\sum_{\mathbf{k}}\nu(a_{\mathbf{k}}{}^{s\dagger}a_{\mathbf{k}}{}^s+\tfrac{1}{2})+\text{same terms with }a\to b.\end{aligned}\qquad(25)$$

It is important to notice that the scalar mesons now have positive energy. The introduction of a negative metric is, of course, a rather drastic measure. However, we regard this only as an artifice to make possible a meaningful discussion of the limit $\xi\to0+$. A consequence of the positive definite H_0 is that the vacuum expectation value of the time ordered product $T[\varphi_\mu(x)\varphi_\nu(0)]$ is given by

$$(2\pi)^{-4}\int S\exp(ip_\mu x_\mu)dp_0d^3p,\qquad(26)$$

where S is given by (17).

4.3 Hamiltonian in Interaction Representation

In the interaction representation, the field operators φ_μ, $\varphi_\mu{}^\star$ have the same space-time dependence as the free case. The metric η remains to be given by (24). In a similar manner to (14) the interaction Hamiltonian H_{int} is given by

$$\begin{aligned}H_{\text{int}}=&-ieA_\mu[g_{\mu\nu}{}^\star\varphi_\nu-g_{\mu\nu}\varphi_\nu{}^\star]-ie\xi A_\mu[\varphi_\mu(\partial\varphi_\nu{}^\star/\partial x_\nu)-\varphi_\mu{}^\star(\partial\varphi_\nu/\partial x_\nu)]+e^2[(A_jA_j)(\varphi_k{}^\star\varphi_k)-(A_j\varphi_j)(A_k\varphi_k{}^\star)]\\&+ie\kappa F_{\mu\nu}\varphi_\mu{}^\star\varphi_\nu+\tfrac{1}{2}e^2\kappa^2[(\varphi_4{}^\star)^2\varphi_j\varphi_j+(\varphi_4)^2\varphi_j{}^\star\varphi_j{}^\star-2(\varphi_4{}^\star\varphi_4)(\varphi_j{}^\star\varphi_j)],\qquad(27)\end{aligned}$$

where

$$g_{\mu\nu}=(\partial\varphi_\nu/\partial x_\mu)-(\partial\varphi_\mu/\partial x_\nu)$$

and

$$g_{\mu\nu}{}^\star=(\partial\varphi_\nu{}^\star/\partial x_\mu)-(\partial\varphi_\mu{}^\star/\partial x_\nu).\qquad(28)$$

The interaction Hamiltonian is not a Hermitian matrix but one that satisfies

$$H_{\text{int}}{}^\star\equiv\eta^{-1}H_{\text{int}}{}^\dagger\eta=H_{\text{int}}.\qquad(29)$$

4.4 Feynman Diagram

In the interaction representation the S matrix for such a theory can be analyzed into sums of Feynman diagrams in exactly the same way as an ordinary theory with positive metric. The values of the propagators and vertices are listed in Fig. 3.

From the rules for Feynman diagram it is clear that the theory satisfies relativistic invariance.

4.5 Unitarity

Because of (29), the S matrix is not unitary but satisfies

$$S^\star\equiv\eta^{-1}S^\dagger\eta=S^{-1}.\qquad(30)$$

If we restrict ourselves to a system of particles with a total energy

$$E<\xi^{-\frac{1}{2}}m,\qquad(31)$$

then by using (25) it is seen that there can be *no* scalar meson in either the initial state or the final state. Thus, for the initial and final states $\eta=+1$ and the S matrix is truly unitary provided (31) holds. Consequently, if

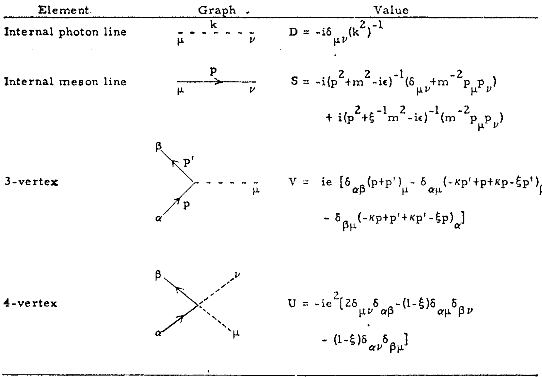


FIG. 3. Feynman diagram in momentum space for Lagrangian (7) with a negative metric (see also the caption of Fig. 1).

the limit $\xi \to 0$ exists, the limiting S matrix does become completely unitary.

4.6 Renormalization

For $\xi > 0$, the propagator of W varies asymptotically like p^{-2} at large momentum. The presence of the ξ-dependent term in the propagator acts like a regulator. Therefore, the divergencies that occur in the higher order Feynman diagrams can be eliminated by a renormalization process which is quite similar to that in the case of a charged scalar meson (except for the differences in the spin dependences).

APPENDICES

In the following appendices we give the detailed derivation of the rules for Feynman graphs for the charged vector mesons following closely the Dyson-Wick procedure.[5] These derivations are at times rather complicated. For clarity we begin with the well-known and almost trivial case of charged vector mesons interacting with the lepton fields.

APPENDIX A. CHARGED VECTOR MESON AND FERMION CURRENTS

We discuss first the derivation of Feynman graph for the simple case of charged vector mesons interacting with electrons and neutrinos.

A1. Lagrangian and Hamiltonian

The Lagrangian density for this case is given by

$$\mathfrak{L} = \mathfrak{L}_W + \mathfrak{L}_{\text{free leptons}} + \mathfrak{L}_1, \tag{A1}$$

where

$$\mathfrak{L}_W = -\tfrac{1}{2} G_{\mu\lambda}{}^* G_{\mu\lambda} - m^2 \Phi_\mu{}^* \Phi_\mu \tag{A2}$$

and

$$\mathfrak{L}_1 = J_\mu \Phi_\mu{}^* + J_\mu{}^* \Phi_\mu, \tag{A3}$$

in which * has the same meaning as that given in (2),

$$G_{\mu\lambda} = (\partial \Phi_\lambda / \partial x_\mu) - (\partial \Phi_\mu / \partial x_\lambda), \tag{A4}$$

$$J_\mu = ig \Psi_e{}^\dagger \gamma_4 \gamma_\lambda (1+\gamma_5) \Psi_\nu, \tag{A5}$$

and Ψ_e, Ψ_ν, Φ_λ are, respectively, the field operators for e^-, ν, and W^+. The dynamic equation for Φ is given by

$$(\partial G_{\mu\lambda}/\partial x_\mu) - m^2 \Phi_\lambda + J_\lambda = 0. \tag{A6}$$

By using $\mathfrak{L}$ one obtains the following Hamiltonian:

$$H = H_W + H_{\text{free lepton}} + H_1, \tag{A7}$$

where

$$H_W = \boldsymbol{\pi}^* \cdot \boldsymbol{\pi} + m^{-2} (\boldsymbol{\nabla} \cdot \boldsymbol{\pi}^*)(\boldsymbol{\nabla} \cdot \boldsymbol{\pi}) + (\boldsymbol{\nabla} \times \boldsymbol{\Phi}^*) \cdot (\boldsymbol{\nabla} \times \boldsymbol{\Phi}) + m^2 \boldsymbol{\Phi}^* \cdot \boldsymbol{\Phi}, \tag{A8}$$

$$H_1 = -J_\mu \Phi_\mu{}^* - J_\mu{}^* \Phi_\mu + m^{-2} J_4 J_4{}^*. \tag{A9}$$

The 3-vectors $\boldsymbol{\pi}$ and $\boldsymbol{\pi}^*$ are, respectively, the conjugate momenta of $\boldsymbol{\Phi}$ and $\boldsymbol{\Phi}^*$

$$\pi_k = i G_{4k}{}^*$$

and

$$\pi_k{}^* = i G_{4k} \quad (k = 1, 2, 3). \tag{A10}$$

Because of the absence of $d\Phi_4/dt$ and $d\Phi_4{}^*/dt$ in $\mathfrak{L}$, the Φ_4 and $\Phi_4{}^*$ in (A9) are not independent variables but are given by

$$\Phi_4 = m^{-2} [i \boldsymbol{\nabla} \cdot \boldsymbol{\pi}^* + J_4],$$

and

$$\Phi_4{}^* = m^{-2} [i \boldsymbol{\nabla} \cdot \boldsymbol{\pi} + J_4{}^*]. \tag{A11}$$

A2. Interaction Representation and Feynman Graphs

In the interaction representation it is convenient to introduce the following notations[10]:

$$\boldsymbol{\varphi} = \boldsymbol{\Phi},$$

$$\varphi_4 = i m^{-2} \boldsymbol{\nabla} \cdot \boldsymbol{\pi}^*,$$

and

$$g_{\mu\lambda} = (\partial \varphi_\lambda / \partial x_\mu) - (\partial \varphi_\mu / \partial x_\lambda). \tag{A12}$$

Therefore, the φ_λ and $g_{\mu\lambda}$ satisfy the free-meson equation

$$\frac{\partial}{\partial x_\mu} g_{\mu\lambda} - m^2 \varphi_\lambda = 0.$$

In terms of φ_λ the interaction Hamiltonian is given by

$$H_{\text{int}} = H_1 = -j_\mu \varphi_\mu{}^* - j_\mu{}^* \varphi_\mu - m^{-2} j_4 j_4{}^*. \tag{A13}$$

Using the notation of Wick,[5] the propagator of $W^\pm$ is given by

$$\varphi_\mu{}^{\cdot}(x) \varphi_\lambda{}^{*\cdot}(0) \equiv \langle T[\varphi_\mu(x) \varphi_\lambda{}^*(0)] \rangle_{\text{vac}}. \tag{A14}$$

Theorem 1.

$$\varphi_\mu{}^{\cdot}(x) \varphi_\lambda{}^{*\cdot}(0) = \mathfrak{D}_{\mu\lambda}(x) + i m^{-2} \delta_{4\mu} \delta_{4\lambda} \delta^4(x), \tag{A15}$$

where

$$\mathfrak{D}_{\mu\lambda}(x) = [\delta_{\mu\lambda} - m^{-2} (\partial^2 / \partial x_\mu \partial x_\lambda)] \tfrac{1}{2} \Delta_F(x), \tag{A16}$$

[10] Throughout all the appendices we use capital letters to denote operators in the Heisenberg representation and small letters such as φ_λ, $g_{\mu\nu}$, a_λ, $f_{\mu\nu}$, j_μ, etc., to denote operators in the interaction representation.

$$\Delta_F(x)=-i(8\pi^4)^{-1}\times\int d^4k\,[k^2+(m-i\epsilon)^2]^{-1}\exp(ik_\lambda x_\lambda),\quad \text{(A17)}$$

$k^2=k_\lambda k_\lambda$, $d^4k=d^3\mathbf{k}(-idk_4)$, and $\delta^4(x)=\delta^3(\mathbf{r})\delta(t)$.

Proof. To avoid possible mathematical ambiguity we shall put the whole system in a finite three-dimensional volume Ω. (The limit $\Omega\to\infty$ will be carried out only at the end.) The field operators φ_μ are then expanded into the following Fourier series:

$$\boldsymbol{\varphi}(x)=\sum_{\mathbf{k}}\sum_{t=1,2}(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^t\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)+b_{-\mathbf{k}}{}^{t\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)]\mathbf{e}_{\mathbf{k}}{}^t+\sum_{\mathbf{k}}(2\Omega\omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^l\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)+b_{-\mathbf{k}}{}^{l\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)](\omega\hat{k}/m)$$

and

$$\varphi_4(x)=\sum_{\mathbf{k}}(2\omega\Omega)^{-\frac{1}{2}}[a_{\mathbf{k}}{}^l\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)-b_{-\mathbf{k}}{}^{l\dagger}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)](i|\mathbf{k}|/m),\quad \text{(A18)}$$

where $\mathbf{e}_{\mathbf{k}}{}^1$, $\mathbf{e}_{\mathbf{k}}{}^2$, and $\hat{k}=|\mathbf{k}|^{-1}\mathbf{k}$ form a right-handed orthogonal set of three unit vectors, $\omega=(\mathbf{k}^2+m^2)^{\frac{1}{2}}>0$ and $a_{\mathbf{k}}{}^r$, $b_{\mathbf{k}}{}^r$ are the annihilation operators for the transverse ($r=t=1, 2$) and the longitudinal ($r=l$) mesons. For definiteness, let us define

$$T[\varphi_\mu(x)\varphi_\nu^*(0)]=\varphi_\mu(x)\varphi_\nu^*(0)\quad\text{if}\quad t\geqq 0,$$

and

$$=\varphi_\nu^*(0)\varphi_\mu(x)\quad\text{if}\quad t<0.\quad \text{(A19)}$$

Therefore, the vacuum expectation value of the T product (A14) is given by (keeping Ω finite)

$$\langle T[\varphi_\mu(x)\varphi_\nu^*(0)]_\Omega\rangle_{\text{vac}}=\sum_{\mathbf{k}}(2\omega\Omega)^{-1}\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)\times[\delta_{\mu\nu}-q_\mu q_\nu m^{-2}]\quad\text{if}\quad t\geqq 0,$$

and

$$=\sum_{\mathbf{k}}(2\omega\Omega)^{-1}\exp(i\mathbf{k}\cdot\mathbf{r}+i\omega t)\times[\delta_{\mu\nu}-q_\mu^* q_\nu^* m^{-2}]\quad\text{if}\quad t<0,$$

where $q_4=i\omega$, $q_j=k_j$ ($j=1, 2, 3$), and q_μ^* is the complex conjugate of q_μ. Upon converting the summand on the right-hand side of the above equation into a Feynman-type integral, we find

$$\langle T[\varphi_\mu(x)\varphi_\nu^*(0)]_\Omega\rangle_{\text{vac}}=-i\sum_{\mathbf{k}}(2\pi\Omega)^{-1}\int_{-\infty}^{\infty}dk_0\left[\frac{\delta_{\mu\nu}-m^{-2}k_\mu k_\nu}{k_\alpha k_\alpha+(m-i\epsilon)^2}-m^{-2}\delta_{4\mu}\delta_{4\nu}\right]\times\exp(ik_\beta x_\beta),\quad \text{(A20)}$$

where $k_4=ik_0$. In (A20) we neglect functions that are zero if $t\neq 0$ and remain finite at $t=0$. Taking the limit $\Omega\to\infty$, we obtain (A15).

It is important to notice that

(i) An expression identical with (A20) would be obtained if instead of (A19) we define $T[\varphi_\mu(x)\varphi_\nu^*(0)]=\varphi_\mu(x)\varphi_\nu^*(0)$ for $t>0$ and $T[\varphi_\mu(x)\varphi_\nu^*(0)]=\varphi_\nu^*(0)\varphi_\mu(x)$ for $t\leqq 0$.

(ii) Because of the usual quantization procedures the limit $\lim_{\Omega\to\infty}\langle T[\varphi_\mu(x)\varphi_\nu^*(0)]_\Omega\rangle_{\text{vac}}$ is not covariant.

(iii) The presence of the term $\delta^4(x)$ in (A15) can also be easily seen by considering the special case $\mu=\lambda=4$. From (A14) and (A11) it follows that $\varphi_4^{\cdot}(x)\varphi_4^{*\cdot}(0)$ is continuous in time at $t=0$. However, $\partial\Delta_F(x)/\partial t$ approaches $-i\delta^3(\mathbf{r})$ at $t=0+$ and $+i\delta^3(\mathbf{r})$ at $t=0-$. Thus $\partial^2[\frac{1}{2}\Delta_F(x)]/\partial t^2$ contains a $\delta^4(x)$ singularity which is to be canceled by the last term on the right-hand side of (A15). Following Dyson's method[5] and by using (A13) and (A15) the S matrix can be evaluated. It can be shown quite easily that in the calculation of S matrix, after converting the appropriate T products into S products, the effects of the contact term $-m^{-2}j_4^*j_4$ in (A13) exactly cancel that of the term $im^{-2}\delta_{4\mu}\delta_{4\nu}\delta^4(x)$ in the propagator (A15). Therefore, one obtains the following theorem:

Theorem 2. The entire S matrix can be generated by considering an equivalent problem in which H_{int} is replaced by[11]

$$H_{\text{int}}'=-j_\mu\varphi_\mu'^*-j_\mu^*\varphi_\mu'\quad \text{(A21)}$$

and the propagator (A15) is replaced by

$$\varphi_\mu'^{\cdot}(x)\varphi_\lambda'^{*\cdot}(0)=[\delta_{\mu\lambda}-m^{-2}(\partial^2/\partial x_\mu\partial x_\lambda)]\tfrac{1}{2}\Delta_F(x).\quad \text{(A22)}$$

Theorem 2 leads to the well-known results first stated by Feynman.[2] The resulting Feynman graphs contain only one kind of vertices which connects two lepton lines and one meson line. In such a graph each internal meson line contributes only the covariant factor

$$-i(\delta_{\mu\lambda}+m^{-2}k_\mu k_\lambda)(k^2+m^2)^{-1},$$

where k_μ is the momentum carried by such a line.

[11] The precise meaning of Theorem 2 is as follows: Regard the S matrix as given by

$$S=\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int T[\prod_{i=1}^{n}H_{\text{int}}'(x_i)d^3r_idt_i].$$

In converting these T products to S products, one uses Theorems 1 and 2 of Wick's paper[5] together with the identity

$$T[\varphi_\mu'(x)\varphi_\nu'^*(y)]=:\varphi_\mu'(x)\varphi_\nu'^*(y):+[\varphi_\mu'(x)]^{\cdot}[\varphi_\nu'^*(y)]^{\cdot},$$

where $[\varphi_\mu'(x)]^{\cdot}[\varphi_\nu'^*(y)]^{\cdot}$ is given by (A22). The resulting S matrix (expressed as a sum of S products of φ_μ' and $\varphi_\nu'^*$) is identical with the original S matrix which is obtained by using H_{int} given by (A13) and $\varphi_\mu^{\cdot}(x)\varphi_\nu^{*\cdot}(y)$ given by (A15).

Exactly the same meaning applies to Theorem 3, and the two lemmas.

APPENDIX B. ELECTRODYNAMICS OF CHARGED VECTOR MESONS

B1. Lagrangian and Hamiltonian

We start with the Lagrangian[10] given by (1),

$$\mathcal{L}=-\tfrac{1}{2}(\partial A_\mu/\partial x_\nu)(\partial A_\mu/\partial x_\nu)-\tfrac{1}{2}G_{\mu\nu}{}^*G_{\mu\nu}-m^2\Phi_\mu{}^*\Phi_\mu-ie\kappa F_{\mu\nu}\Phi_\mu{}^*\Phi_\nu,$$

and regard Φ_4, $\Phi_4{}^*$ as dependent variables.

Let $\boldsymbol{\pi}$, $\boldsymbol{\pi}^*$, $\mathbf{P}$, P_0 be, respectively, the conjugate momenta of $\boldsymbol{\Phi}$, $\boldsymbol{\Phi}^*$, $\mathbf{A}$, and $A_0(=-iA_4)$. We have

$$\pi_k=iG_{4k}{}^*,$$
$$\pi_k{}^*=iG_{4k},$$
$$\mathbf{P}=(d\mathbf{A}/dt)-e\kappa(\Phi_4{}^*\boldsymbol{\Phi}-\boldsymbol{\Phi}^*\Phi_4),$$

and

$$P_0=-dA_0/dt, \tag{A23}$$

where $k=1, 2, 3$. The Hamiltonian density H is given by[6]

$$\begin{aligned}H=&\tfrac{1}{2}(\mathbf{P}^2-P_0{}^2)+\tfrac{1}{2}(\partial A_\mu/\partial x_i)(\partial A_\mu/\partial x_i)+\tfrac{1}{2}G_{jk}{}^*G_{jk}\\&+\boldsymbol{\pi}^*\cdot\boldsymbol{\pi}+m^2(\boldsymbol{\Phi}^*\cdot\boldsymbol{\Phi}+\Phi_4{}^*\Phi_4)\\&+i(\boldsymbol{\pi}\cdot\boldsymbol{\nabla}\Phi_4+\boldsymbol{\pi}^*\cdot\boldsymbol{\nabla}\Phi_4{}^*)+e(\boldsymbol{\pi}^*\cdot\boldsymbol{\Phi}^*-\boldsymbol{\pi}\cdot\boldsymbol{\Phi})A_4\\&+e(\Phi_4\boldsymbol{\pi}-\Phi_4{}^*\boldsymbol{\pi}^*)\cdot\mathbf{A}+e\kappa(\mathbf{P}-ie\boldsymbol{\nabla}A_4)(\Phi_4{}^*\boldsymbol{\Phi}-\Phi_4\boldsymbol{\Phi}^*)\\&+ie\kappa F_{jk}\Phi_j{}^*\Phi_k+\tfrac{1}{2}e^2\kappa^2(\Phi_4{}^*\boldsymbol{\Phi}-\Phi_4\boldsymbol{\Phi}^*)^2.\end{aligned} \tag{A24}$$

In the Hamiltonian both Φ_4 and $\Phi_4{}^*$ are regarded as functions of $\boldsymbol{\Phi}$, $\mathbf{A}$, A_4, $\boldsymbol{\pi}$, $\mathbf{P}$, etc. By using (5) and (A23) we obtain

$$D\Phi_4=[1-(e\kappa/m)^2\boldsymbol{\Phi}^*\cdot\boldsymbol{\Phi}]N-(e\kappa/m)^2\boldsymbol{\Phi}\cdot\boldsymbol{\Phi}N^*,$$
$$D\Phi_4{}^*=[1-(e\kappa/m)^2\boldsymbol{\Phi}^*\cdot\boldsymbol{\Phi}]N^*-(e\kappa/m)^2\boldsymbol{\Phi}^*\cdot\boldsymbol{\Phi}^*N,$$

where

$$m^{-2}D=[1-(e\kappa/m)^2\boldsymbol{\Phi}^*\cdot\boldsymbol{\Phi}]^2-(e\kappa/m)^4(\boldsymbol{\Phi}^*\cdot\boldsymbol{\Phi}^*)(\boldsymbol{\Phi}\cdot\boldsymbol{\Phi}),$$
$$N=i\boldsymbol{\nabla}\cdot\boldsymbol{\pi}^*+e\mathbf{A}\cdot\boldsymbol{\pi}^*-e\kappa\boldsymbol{\Phi}\cdot(\mathbf{P}-i\boldsymbol{\nabla}A_4),$$

and

$$N^*=i\boldsymbol{\nabla}\cdot\boldsymbol{\pi}-e\mathbf{A}\cdot\boldsymbol{\pi}+e\kappa\boldsymbol{\Phi}^*\cdot(\mathbf{P}-i\boldsymbol{\nabla}A_4). \tag{A25}$$

B2. Interaction Representation

In a similar manner to (A12) it is convenient to introduce the following notations in the interaction representation[10]:

$$\boldsymbol{\varphi}=\boldsymbol{\Phi}$$
$$\varphi_4=im^{-2}\boldsymbol{\nabla}\cdot\boldsymbol{\pi}^*$$

and

$$a_\mu=A_\mu. \tag{A26}$$

These field operators φ_μ, a_μ, therefore, satisfy the free-meson and the free-photon equations,

$$(\partial g_{\mu\nu}/\partial x_\mu)-m^2\varphi_\nu=0$$

and

$$(\partial^2 a_\nu/\partial x_\lambda\partial x_\lambda)=0, \tag{A27}$$

where

$$g_{\mu\nu}=(\partial\varphi_\nu/\partial x_\mu)-(\partial\varphi_\mu/\partial x_\nu). \tag{A28}$$

In terms of φ_μ, a_μ the interaction Hamiltonian is given by

$$\begin{aligned}H_{\rm int}=&-iea_\mu[g_{\mu\nu}{}^*\varphi_\nu-g_{\mu\nu}\varphi_\nu{}^*]\\&+e^2[(\mathbf{a}\times\boldsymbol{\varphi}^*)\cdot(\mathbf{a}\times\boldsymbol{\varphi})-(a_jg_{4j}{}^*)(a_kg_{4k})m^{-2}]\\&+ie\kappa f_{jk}\varphi_j{}^*\varphi_k+ie\kappa f_{4j}(\Phi_4{}^*\varphi_j-\varphi_j{}^*\Phi_4)\\&+\tfrac{1}{2}e^2\kappa^2(\Phi_4{}^*\boldsymbol{\varphi}-\boldsymbol{\varphi}^*\Phi_4)^2+m^2y^*y,\end{aligned} \tag{A29}$$

where

$$f_{\mu\nu}=(\partial a_\nu/\partial x_\mu)-(\partial a_\mu/\partial x_\nu), \tag{A30}$$
$$y=\Phi_4-\varphi_4-im^{-2}ea_jg_{4j},$$

and

$$y^*=\Phi_4{}^*-\varphi_4{}^*+im^{-2}ea_jg_{4j}. \tag{A31}$$

In the above, Φ_4 and $\Phi_4{}^*$ are regarded as functions of φ_μ, a_μ, and their derivatives. The explicit forms of the functions Φ_4 and $\Phi_4{}^*$ can be directly obtained by using (A25), (A26), and the following substitutions:

$$\pi_j=ig_{4j}{}^*,\quad \pi_j{}^*=ig_{4j},$$

and

$$[P_j-i(\partial a_4/\partial x_j)]=if_{4j}. \tag{A32}$$

B3. Feynman Graphs

To obtain the appropriate rules for Feynman graphs we adopt the procedures and the notations used in Wick's paper.[5] The contraction of any two operators $A(x)$ and $B(y)$ is defined to be the vacuum expectation of their T product in the interaction representation:

$$A^\cdot(x)B^\cdot(y)\equiv\langle T[A(x)B(y)]\rangle_{\rm vac}. \tag{A33}$$

In a similar manner to (A15), many of the contractions between the operators φ_μ, $g_{\mu\nu}$, etc. cannot be expressed in terms of covariant functions. By using (A28), (A30), and (A32) we obtain the following noncovariant contractions:

$$\varphi_\mu{}^\cdot(x)\varphi_\nu{}^{*\cdot}(y)=\mathfrak{D}_{\mu\nu}(x-y)+im^{-2}\delta_{4\mu}\delta_{4\nu}\delta^4(x-y),$$

$$g_{\mu\nu}{}^\cdot(x)g_{\alpha\beta}{}^{*\cdot}(y)=-\frac{\partial^2}{\partial x_\mu\partial x_\alpha}\mathfrak{D}_{\nu\beta}-\frac{\partial^2}{\partial x_\nu\partial x_\beta}\mathfrak{D}_{\mu\alpha}+\frac{\partial^2}{\partial x_\nu\partial x_\alpha}\mathfrak{D}_{\mu\beta}+\frac{\partial^2}{\partial x_\mu\partial x_\beta}\mathfrak{D}_{\nu\alpha}+i[\delta_{4\mu}\delta_{4\alpha}\delta_{\nu\beta}+\delta_{4\nu}\delta_{4\beta}\delta_{\mu\alpha}-\delta_{4\mu}\delta_{4\beta}\delta_{\nu\alpha}-\delta_{4\nu}\delta_{4\alpha}\delta_{\mu\beta}]\delta^4(x-y)$$

and

$$f_{\mu\nu}{}^{\cdot}(x)f_{\alpha\beta}{}^{\cdot}(y)=\left[-\delta_{\nu\beta}\frac{\partial^2}{\partial x_\mu\partial x_\alpha}-\delta_{\mu\alpha}\frac{\partial^2}{\partial x_\nu\partial x_\beta}+\delta_{\mu\beta}\frac{\partial^2}{\partial x_\nu\partial x_\alpha}+\delta_{\nu\alpha}\frac{\partial^2}{\partial x_\mu\partial x_\beta}\right]\tfrac{1}{2}D_F(x-y)$$
$$+i[\delta_{4\mu}\delta_{4\alpha}\delta_{\nu\beta}+\delta_{4\nu}\delta_{4\beta}\delta_{\mu\alpha}-\delta_{4\mu}\delta_{4\beta}\delta_{\nu\alpha}-\delta_{4\nu}\delta_{4\alpha}\delta_{\mu\beta}]\delta^4(x-y), \quad \text{(A34)}$$

where $\mathfrak{D}_{\mu\nu}(x-y)$ is given by (A16), Δ_F by (A17), and

$$D_F(x)=-i(8\pi^4)^{-1}\int d^4k(k^2)^{-1}\exp(ik_\lambda x_\lambda). \quad \text{(A35)}$$

The other relevant contractions can all be expressed in covariant forms; for example,

$$a_\mu{}^{\cdot}(x)a_\nu{}^{\cdot}(y)=\tfrac{1}{2}\delta_{\mu\nu}D_F(x-y),$$
$$g_{\mu\nu}{}^{\cdot}(x)\varphi_\lambda{}^{*\cdot}(y)=(\partial/\partial x_\mu)\mathfrak{D}_{\nu\lambda}(x-y)-(\partial/\partial x_\nu)\mathfrak{D}_{\mu\lambda}(x-y), \quad \text{(A36)}$$

etc.

It is important to notice that if $\kappa\neq0$ the expansion of H_{int} (A29) into a power series of e actually contains an infinite number of terms. In principle, the rules for Feynman graphs can be obtained by a straight-forward application of the standard algebraic method[5] of converting the T products in the S matrix into the appropriate S products. In practice, because of the complexity of H_{int} and the presence of numerous noncovariant terms in the propagators, it is quite complicated to carry out the details. This will be done in Appendix C. It is found that, similar to the simple example of the interaction between vector mesons and lepton currents discussed in Appendix A, much of these two above mentioned complexities cancel themselves. We obtain, as a result, the following theorem (proved in Appendix C).

Theorem 3. The above S matrix can be generated by considering an equivalent problem in which H_{int} is replaced by[11]

$$H_{\text{int}}'=-iea_\mu'[g_{\mu\nu}'^*\varphi_\nu'-g_{\mu\nu}'\varphi_\nu'^*]+e^2a_\mu'a_\nu'[\delta_{\mu\nu}\varphi_\lambda'^*\varphi_\lambda'-\varphi_\mu'^*\varphi_\nu']+ie\kappa f_{\mu\nu}'\varphi_\mu'^*\varphi_\nu'+\delta H, \quad \text{(A37)}$$

where

$$\delta H=(i/2)\delta^4(0)\ln\{[1-(e\kappa/m)^2\varphi_j'^*\varphi_j']^2-(e\kappa/m)^4(\varphi_j'\varphi_j')(\varphi_k'^*\varphi_k'^*)\}. \quad \text{(A38)}$$

The contraction of the prime fields φ_μ', $g_{\mu\nu}'$, etc., are identical with that of φ_μ, $g_{\mu\nu}$, etc., *except* that all the noncovariant terms are now absent. More explicitly, (A34) is replaced by

$$\varphi_\mu'{}^{\cdot}(x)\varphi_\nu'{}^{*\cdot}(y)=\mathfrak{D}_{\mu\nu}(x-y),$$

$$g_{\mu\nu}'{}^{\cdot}(x)g_{\alpha\beta}'{}^{*\cdot}(y)=-\frac{\partial^2}{\partial x_\mu\partial x_\alpha}\mathfrak{D}_{\nu\beta}-\frac{\partial^2}{\partial x_\nu\partial x_\beta}\mathfrak{D}_{\mu\alpha}+\frac{\partial^2}{\partial x_\nu\partial x_\alpha}\mathfrak{D}_{\mu\beta}+\frac{\partial^2}{\partial x_\mu\partial x_\beta}\mathfrak{D}_{\nu\alpha}, \quad \text{(A39)}$$

etc., and (A36) remains the same as before; i.e.,

$$a_\mu'{}^{\cdot}(x)a_\nu'{}^{\cdot}(y)=\tfrac{1}{2}\delta_{\mu\nu}D_F(x-y), \quad \text{(A40)}$$

etc.

Theorem 3 states that except for the term δH in (A37) the effects of the noncovariant terms in the original propagators (A34) completely cancel those that are generated by the difference between H_{int} and $-\mathfrak{L}_{\text{int}}$. If $\kappa=0$, $\delta H=0$; therefore, by using Theorem 3, the rules of deriving Feynman diagrams becomes almost trivial. However, if $\kappa\neq0$, (δH) gives rise to additional vertices which are both divergent and noncovariant.

These results are summarized in Fig. 1.

APPENDIX C. PROOF OF THEOREM 3

C1. A Simple System of Harmonic Oscillators

Let us consider a problem of N harmonic oscillators whose coordinates are Q_1, $Q_2\cdots$, Q_n and frequencies $\omega_1=\omega_2=\cdots=1$. The Lagrangian for this system is given by

$$L=L_0+L_1, \quad \text{(A41)}$$

where

$$L_0=\frac{1}{2}\frac{d\tilde{Q}}{dt}\frac{dQ}{dt}-\tfrac{1}{2}\tilde{Q}Q,$$

$$L_1=\frac{1}{2}\frac{d\tilde{Q}}{dt}A\frac{dQ}{dt}+\frac{1}{2}\frac{d\tilde{Q}}{dt}B+\tfrac{1}{2}\tilde{B}\frac{dQ}{dt}+C,$$

$$Q=\begin{bmatrix}Q_1\\Q_2\\\vdots\\Q_N\end{bmatrix}, \quad \text{(A42)}$$

$\tilde{Q}$ is the transpose of Q, A is a symmetric $(N\times N)$ matrix and B, C are, respectively, matrices of dimension $(N\times1)$ and (1×1). All three matrices A, B, C are *functions* of Q (but do not explicitly depend on dQ/dt). The conjugate momenta P and the Hamiltonian H are given by

$$P=(1+A)(dQ/dt)+B$$

and

$$H=\tfrac{1}{2}\tilde{P}(1+A)^{-1}P+\tfrac{1}{2}\tilde{Q}Q-\tfrac{1}{2}\tilde{P}(1+A)^{-1}B-\tfrac{1}{2}\tilde{B}(1+A)^{-1}P-C+\tfrac{1}{2}\tilde{B}(1+A)^{-1}B, \quad \text{(A43)}$$

where 1 is the $(N\times N)$ unit matrix.

In the following we discuss the perturbation series in which A, B, C are treated as small but arbitrary functions of Q. It is convenient to use the interaction representation, regarding

$$H_1\equiv H-\tfrac{1}{2}\tilde{P}P-\tfrac{1}{2}\tilde{Q}Q \quad \text{(A44)}$$

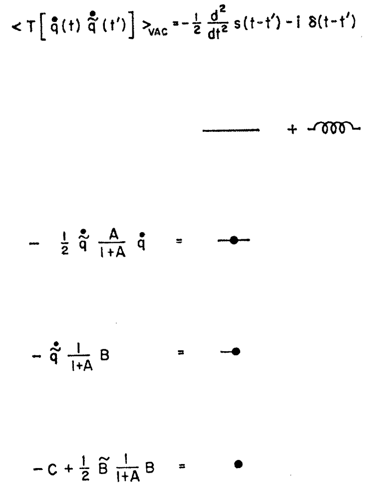


FIG. 4. Propagator and vertices for the simple system discussed in Appendix C2. The dot over $q(t)$ denotes a time derivative.

as the interaction Hamiltonian. For clarity, we introduce in the interaction representation[10]

$$q \equiv \begin{bmatrix} q_1 \\ \vdots \\ q_N \end{bmatrix} \equiv Q.$$

Therefore,

$$dq/dt = P. \tag{A45}$$

The explicit time dependences of q and dq/dt are given by

$$q_n = (1/\sqrt{2})(a_n e^{-it} + a_n^\dagger e^{it})$$

and

$$dq_n/dt = -(i/\sqrt{2})(a_n e^{-it} - a_n^\dagger e^{it}), \tag{A46}$$

where a_n and $a_n^\dagger$ are, respectively, the annihilation and creation operators for the nth harmonic oscillator ($n=1, 2, \cdots N$). In terms of q and dq/dt the interaction Hamiltonian H_1 becomes

$$H_1 = -\frac{1}{2}\frac{d\tilde{q}}{dt}A(1+A)^{-1}\frac{dq}{dt} - \frac{1}{2}\frac{d\tilde{q}}{dt}(1+A)^{-1}B$$
$$-\tfrac{1}{2}\tilde{B}(1+A)^{-1}\frac{dq}{dt} - C + \tfrac{1}{2}\tilde{B}(1+A)^{-1}B. \tag{A47}$$

We observe that the vacuum expectation values of the various T products in the interaction representation are given by

$$\langle T[q_n(t)q_m(0)]\rangle_{\text{vac}} = \tfrac{1}{2}\delta_{nm}s(t),$$
$$\langle T[(dq_n/dt)(t)q_m(0)]\rangle_{\text{vac}} = \tfrac{1}{2}\delta_{nm}ds/dt, \tag{A48}$$

and

$$\left\langle T\left[\frac{dq_n}{dt}(t)\frac{dq_m}{dt}(0)\right]\right\rangle_{\text{vac}} = \tfrac{1}{2}\delta_{nm}s$$
$$= -\tfrac{1}{2}\delta_{nm}\frac{d^2s}{dt^2} - i\delta_{nm}\delta(t), \tag{A49}$$

where n and m vary from 1 to N and

$$s(t) = \exp(-it) \quad \text{for} \quad t \geqq 0$$
$$= \exp(it) \quad \text{for} \quad t \leqq 0. \tag{A50}$$

As in the previous case of vector mesons, (A49) states that the contraction between time derivatives of q_n and q_m differs from the time derivatives of the corresponding contraction. We now state the following lemma:

Lemma 1. The S matrix of this problem can be generated by considering an equivalent interaction Hamiltonian[11]

$$H_1' = -\frac{1}{2}\left(\frac{d}{dt}\tilde{q}'\right)A\left(\frac{d}{dt}q'\right) - \frac{1}{2}\left(\frac{d}{dt}\tilde{q}'\right)B$$
$$-\tfrac{1}{2}\tilde{B}\left(\frac{d}{dt}q'\right) - C + \delta H, \tag{A51}$$

where $\delta H = \frac{1}{2}i\delta(0)$ trace$[\ln(1+A)]$ and A, B, C are the same functions as before (but replacing q by q'). The contractions between q' and dq'/dt are given by

$$[q_n'(t)]^{\cdot}[q_m'(0)]^{\cdot} = \tfrac{1}{2}\delta_{nm}s(t),$$
$$\left[\frac{dq_n'}{dt}(t)\right]^{\cdot}[q_m'(0)]^{\cdot} = \tfrac{1}{2}\delta_{nm}\frac{ds}{dt},$$

and

$$\left[\frac{dq_n'}{dt}(t)\right]^{\cdot}\left[\frac{dq_m'}{dt}(0)\right]^{\cdot} = -\tfrac{1}{2}\delta_{nm}\frac{d^2s}{dt^2}. \tag{A52}$$

It is important to notice that the term $-i\delta_{nm}\delta(t)$ in (A49) is omitted in (A52) and that H_1' is essentially the same function as $-L_1$ except for the extra term $\frac{1}{2}i\delta(0)$ trace$[\ln(1+A)]$.

C2. Proof of Lemma 1

To prove the lemma we consider the usual power series expansion of the S matrix in the interaction representation

$$S = \sum_{n=0}^{\infty} S_n,$$

where

$$S_n = \frac{(-i)^n}{n!}\int T\left[\prod_{i=1}^{n} H_1(t_i)dt_i\right]. \tag{A53}$$

In converting the above S matrix from T products into S products, let us concentrate on the developments due to the conversion of $T[[dq_n(t)/dt]_t[dq_m(t)/dt]_{t'}]$:

$$T\left[\frac{dq_n}{dt}(t)\frac{dq_m}{dt}(0)\right] = :\frac{dq_n}{dt}(t)\frac{dq_m}{dt}(0):$$
$$+\left\langle T\left[\frac{dq_n}{dt}(t)\frac{dq_m}{dt}(0)\right]\right\rangle_{\text{vac}}.$$

We use the graphical method that only the contraction between dq_n/dt and dq_m/dt is represented, but all other contractions such as $[q_n(t)]^{\cdot}[q_m(0)]^{\cdot}$ and $[dq_n(t)/dt]_t^{\cdot}[q_m(0)]^{\cdot}$ are suppressed (i.e., not represented explicitly in the graphs). In Fig. 4 the propagator $[dq_n(t)/dt]_t^{\cdot}[dq_m(t)/dt]_{t'}^{\cdot}$ is represented by two terms: a straight line which stands for $-\frac{1}{2}\delta_{nm}d^2s/dt^2$ and a "spring" for the second term $-i\delta_{nm}\delta(t)$ in (A49). There are three kinds of vertices which correspond, respectively, to the terms $-\frac{1}{2}(d\tilde{q}/dt)A(1+A)^{-1}(dq/dt)$, $-(d\tilde{q}/dt)(1+A)^{-1}B$, and $[-C+\frac{1}{2}\tilde{B}(1+A)^{-1}B]$. The present problem then reduces simply to one of summing over all diagrams which contain different numbers of springs but otherwise are of similar topological structures. These sums are illustrated in Fig. 5.

To understand the sum I in Fig. 5, let us define

$$\mathrm{I}_1=-\frac{1}{2}\frac{d\tilde{q}}{dt}A(1+A)^{-1}\frac{dq}{dt},$$

which contributes a term $(-i)\int \mathrm{I}_1 dt$ in S_1. In S_2 there is a corresponding term $-i\int \mathrm{I}_2 dt$ that arises from the following contraction:

$$[(-i)^2/2!]\int dtdt'\left\{-\frac{1}{2}\frac{d\tilde{q}}{dt}A(1+A)^{-1}\left[\frac{dq}{dt}(t)\right]^{\cdot}\right\}\times\left\{-\frac{1}{2}\left[\frac{d\tilde{q}}{dt}(t')\right]^{\cdot}A(1+A)^{-1}\frac{dq}{dt}\right\},$$

in which one substitutes only the $-i\delta(t-t')$ part of (A49) for $[dq(t)/dt]_t^{\cdot}[d\tilde{q}(t)/dt]_{t'}^{\cdot}$. There are altogether four such terms due to the four different ways of selecting $(dq_i/dt)(dq_j/dt)$ out of the product $[dq_n(t)/dt]_t\times[dq_m(t)/dt]_t[dq_{n'}(t)/dt]_{t'}[dq_{m'}(t)/dt]_{t'}$. Thus we find

$$\mathrm{I}_2=-\frac{1}{2}\frac{d\tilde{q}}{dt}\left(\frac{A}{1+A}\right)^2\frac{dq}{dt}.$$

Similarly, it is easy to prove that there is a corresponding term $(-i)\int \mathrm{I}_n dt$ in S_n, where

$$\mathrm{I}_n=-\frac{1}{2}\frac{d\tilde{q}}{dt}\left[\frac{A}{1+A}\right]^n\frac{dq}{dt}.$$

The total sum of all these diagrams is given by

$$\mathrm{I}=\sum_{n=1}^{\infty}\mathrm{I}_n=-\frac{1}{2}\frac{d\tilde{q}}{dt}A\frac{dq}{dt},\qquad\text{(A54)}$$

which contributes a term $(-i)\int \mathrm{I}dt$ to the entire S matrix. Identical arguments hold for cases in which I appears only as a part of a bigger diagram. The result of eliminating $-i\delta(t)$ term in the propagators in I-type diagrams is simply to replace $-\frac{1}{2}(d\tilde{q}/dt)A(1+A)^{-1}(dq/dt)$ in the interaction Hamiltonian by $-\frac{1}{2}(d\tilde{q}/dt)A(dq/dt)$.

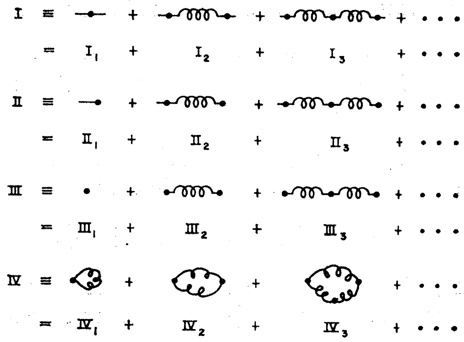


FIG. 5. Sums of certain diagrams discussed in Appendix C2.

To understand the sum II in Fig. 5, let us consider the term $(-i)\int \mathrm{II}_1 dt$ in S_1, where

$$\mathrm{II}_1=-\frac{d\tilde{q}}{dt}(1+A)^{-1}B.$$

By using almost identical arguments as that used in the sum I, it is easy to show that there is also a corresponding term $(-i)\int \mathrm{II}_n dt$ in S_n, where

$$\mathrm{II}_n=-(d\tilde{q}/dt)(1+A)^{-1}[A/(1+A)]^{n-1}B.\qquad\text{(A55)}$$

Thus, summing over n we obtain

$$\mathrm{II}=\sum_{n=1}^{\infty}\mathrm{II}_n=-\frac{d\tilde{q}}{dt}B.\qquad\text{(A56)}$$

In the sum III, the term III_1 is given by

$$\mathrm{III}_1=-C+\tfrac{1}{2}\tilde{B}(1+A)^{-1}B,\qquad\text{(A57)}$$

which contributes a term $(-i)\int \mathrm{III}_1 dt$ to S_1. In S_2, let us consider

$$\frac{(-i)^2}{2!}\int\left\{-\left[\frac{d\tilde{q}}{dt}(t)\right]^{\cdot}(1+A)^{-1}B\right\}\times\left\{-\left[\frac{d\tilde{q}}{dt}(t')\right]^{\cdot}(1+A)^{-1}B\right\}dtdt',$$

and again substitute only the $-i\delta(t-t')$ term for the contraction. The result gives a term $(-i)\int \mathrm{III}_2 dt$ in S_2, where

$$\mathrm{III}_2=-\tfrac{1}{2}\tilde{B}(1+A)^{-2}B.\qquad\text{(A58)}$$

Similarly, the diagram III_n contributes to S_n a term $(-i)\int \mathrm{III}_n dt$, where

$$\mathrm{III}_n=-\tfrac{1}{2}\tilde{B}(1+A)^{-n}A^{n-2}B\quad(n\geqq 2).\qquad\text{(A59)}$$

Summing over n, we obtain

$$\mathrm{III}=\sum_{n=1}^{\infty}\mathrm{III}_n=-C.\qquad\text{(A60)}$$

To understand IV, let us consider in S_1 the contribution of

$$(-i)\int -\frac{1}{2}\left[\frac{d\tilde{q}}{dt}(t)\right]^{\cdot} A(1+A)^{-1}\left[\frac{dq}{dt}(t)\right]^{\cdot} dt$$

in which only $[-i\delta_{nm}\delta(t)]$ is used for the contraction. This results a term $(-i)\int \mathrm{IV}_1 dt$ in S_1, where

$$\mathrm{IV}_1 = +\tfrac{1}{2}i\delta(0)\ \mathrm{trace}[A/(1+A)]. \quad \text{(A61)}$$

Similarly, it can be shown that the diagram IV_n in Fig. 5 contributes a term $(-i)\int \mathrm{IV}_n dt$ to S_n where

$$\mathrm{IV}_n = +\tfrac{1}{2}i\delta(0) n^{-1}\ \mathrm{trace}[A/(1+A)]^n. \quad \text{(A62)}$$

The factor n^{-1} is due to the cyclic symmetry of the diagram IV_n. Summing over n, one obtains

$$\mathrm{IV} = \sum_{n=1}^{\infty} \mathrm{IV}_n = \tfrac{1}{2}i\delta(0)\ \mathrm{trace}[\ln(1+A)]. \quad \text{(A63)}$$

It is easy to see that identical sums can be performed for any part of an arbitrary diagram in which $-i\delta_{nm}\delta(t)$ occurs in the contraction $[dq_n(t)/dt]_t^{\cdot}[dq_m(t)/dt]_0^{\cdot}$. The result of such sum is Lemma 1. We recall that, since A is a function of q, IV is not a constant.

Both Theorems 2 and 3 are direct consequences of this lemma. The last term $\frac{1}{2}i\delta(0)$ trace$[\ln(1+A)]$ in the lemma is the cause of the existence of (δH) in Theorem 3. This is connected with the fact that the dependence of φ_4 on $i\nabla\cdot\pi^*$ makes the extra-magnetic moment term $ie\kappa F_{\mu\nu}\varphi_\mu{}^*\varphi_\nu$ to behave like $-\frac{1}{2}(d\tilde{q}/dt)A(dq/dt)$ in the lemma. The detailed steps leading from Lemma 1 to Theorem 3 are still somewhat involved and are given in the subsequent sections.

C3. Generalization of Lemma 1

The case of vector mesons discussed in Theorem 3 differs from the problem of harmonic oscillators treated in Lemma 1 in several essential aspects. Comparison between (A26) and (A45) suggests that φ_4 and $\varphi_4{}^*$ of the vector mesons fields behave like dq/dt of the harmonic oscillators. Yet, two main differences exist:

(i) The noncovariant term $im^{-2}\delta_{4\mu}\delta_{4\nu}\delta^4(x)$ in $\varphi_\mu{}^{\cdot}(x) \times \varphi_\nu{}^{*\cdot}(0)$ [given by (A34)] does not exactly correspond to the term $-i\delta_{nm}\delta(t)$ in $[dq_n(t)/dt]_t^{\cdot}[dq_m(t)/dt]_0^{\cdot}$ which is given by (A49).

(ii) In Lemma 1, $(H_1'-\delta H)$ is the same function as $-L_1$ if one replaces dq/dt in $(-L_1)$ by dq'/dt and q by q'. The analogy between φ_4, $\varphi_4{}^*$ and dq/dt might suggest that in the case of vector mesons one could first regard $-\mathcal{L}_{\mathrm{int}}$ as a function of φ_μ, $\varphi_\mu{}^*$ through the relations $\Phi_4=\Phi_4(\varphi_\mu,\varphi_\mu{}^*,\cdots)$ and $\Phi_4{}^*=\Phi_4{}^*(\varphi_\mu,\varphi_\mu{}^*,\cdots)$ and then replace in $-\mathcal{L}_{\mathrm{int}}$ all φ_μ, $\varphi_\mu{}^*$ by $\varphi_\mu{}'$ and $\varphi_\mu{}'$. The resulting function would, however, be completely different from $(H_{\mathrm{int}}'-\delta H)$ given by (A37). Rather, Theorem 3 states that $(H_{\mathrm{int}}'-\delta H)$ is the same function as $-\mathcal{L}_{\mathrm{int}}$ only if in $(-\mathcal{L}_{\mathrm{int}})$ the variables Φ_μ and $\Phi_\mu{}^*$ are replaced directly by $\varphi_\mu{}'$ and $\varphi_\mu{}'^*$. (The same is true also for Theorem 2 for which the term corresponds to $\delta H=0$.)

Because of the above differences between φ_4 and dq/dt, Lemma 1 has to be generalized.

Consider a problem in which the interaction Hamiltonian is given by

$$H_{\mathrm{int}} = -\tfrac{1}{2}\tilde{\psi}A(1+A)^{-1}\psi - \tfrac{1}{2}\tilde{\psi}(1+A)^{-1}B - \tfrac{1}{2}\tilde{B}(1+A)^{-1}\psi - C + \tfrac{1}{2}\tilde{B}(1+A)^{-1}B, \quad \text{(A64)}$$

where ψ consists of N local Hermitian operators:

$$\psi = \begin{bmatrix} \psi_1(x) \\ \psi_2(x) \\ \vdots \\ \psi_N(x) \end{bmatrix}, \quad \text{(A65)}$$

$A(x)$ is a symmetric $(N\times N)$ matrix, B is a matrix of dimension $(N\times 1)$ and C is (1×1). The matrix elements of A, B, C are local operators. Let $M(x-y)$ be the contraction between $\psi(x)$ and $\psi(y)$,

$$\psi^{\cdot}(x)\tilde{\psi}^{\cdot}(y) = M(x-y). \quad \text{(A66)}$$

The following lemma can then be established.[11]

Lemma 2. The S matrix of the above problem can also be generated by considering an alternative problem in which (i) the H_{int} in (A64) is replaced by

$$H_{\mathrm{int}}' = -\tfrac{1}{2}\tilde{\psi}'A\psi' - \tfrac{1}{2}\tilde{\psi}'B - \tfrac{1}{2}\tilde{B}\psi' - C + \delta H, \quad \text{(A67)}$$

where

$$\delta H = \tfrac{1}{2}i\delta^4(0)\ \mathrm{trace}[\ln(1+A)], \quad \text{(A68)}$$

and (ii) the contraction (A66) is replaced by

$$\psi'^{\cdot}(x)\tilde{\psi}'^{\cdot}(y) = M(x-y) + i\delta^4(x-y). \quad \text{(A69)}$$

All other contractions such as that between ψ and A, B, C remain unchanged, except for the formal replacement of ψ by ψ'.

Proof. The proof of Lemma 1 can be used directly to prove Lemma 2 by simply changing dq/dt into ψ.

It is useful to observe that the functions $H_{\mathrm{int}}(\psi)$ and $H_{\mathrm{int}}'(\psi')$ are connected by a simple transformation similar to the usual Legendre transformation relating Lagrangian to Hamiltonian. Define

$$G(\psi') \equiv -H_{\mathrm{int}}'(\psi') + \delta H = \tfrac{1}{2}\tilde{\psi}'A\psi' + \tilde{B}\psi' + C \quad \text{(A70)}$$

and

$$\psi_a \equiv \psi_a{}' + \frac{\partial G}{\partial \psi_a{}'} \quad (a=1,2,\cdots N). \quad \text{(A71)}$$

In (A70) and (A71) ψ and ψ' are considered to be c-number vectors. The function $H_{\mathrm{int}}(\psi)$ is, then, given by

$$H_{\mathrm{int}}(\psi) = -\tfrac{1}{2}\sum_{a=1}^{N}\left(\frac{\partial G}{\partial \psi_a{}'}\right)^2 - G(\psi'). \quad \text{(A72)}$$

C4. Proof of Theorem 3

In a similar manner to (A70), let us define

$$G \equiv -H_{\text{int}}' + \delta H, \tag{A73}$$

where H_{int}' is given by (A37). Let us formally regard in (A73) f_{4j}', φ_4', $\varphi_4'^*$, g_{4j}', $g_{4j}'^*$ as 11 independent c-number variables and all others such as $\boldsymbol{\varphi}'$, $\boldsymbol{\varphi}'^*$, g_{ij}', $g_{ij}'^*$, etc. as *constants*. In terms of these 11 variables the function G becomes

$$\begin{aligned} G(f_{4j}',\varphi_4',\varphi_4'^*,g_{4j}',g_{4j}'^*) &= [-(e^2\mathbf{a}\cdot\mathbf{a})\varphi_4'^*\varphi_4' + (iea_j)g_{4j}'\varphi_4'^* \\ &\quad -(iea_j)g_{4j}'^*\varphi_4' + (ie\kappa\varphi_j^*)f_{4j}'\varphi_4' - (ie\kappa\varphi_j)f_{4j}'\varphi_4'^*] \\ &\quad +[(e^2a_4\mathbf{a}\cdot\boldsymbol{\varphi}^*)\varphi_4' + (e^2a_4\mathbf{a}\cdot\boldsymbol{\varphi})\varphi_4'^* \\ &\quad -(iea_4\varphi_j^*)g_{4j}' + (iea_4\varphi_j)g_{4j}^*] + C, \end{aligned} \tag{A74}$$

where C is a constant given by

$$\begin{aligned} C &= iea_i[g_{ij}^*\varphi_j - g_{ij}\varphi_j^*] - e^2a_4^2\boldsymbol{\varphi}\cdot\boldsymbol{\varphi}^* \\ &\quad - e^2(\mathbf{a}\times\boldsymbol{\varphi})\cdot(\mathbf{a}\times\boldsymbol{\varphi}^*) - ief_{ij}\varphi_i^*\varphi_j. \end{aligned} \tag{A75}$$

In both (A74) and (A75) the values of the "constants" $\boldsymbol{\varphi}'$, $\boldsymbol{\varphi}'^*$, g_{ij}', $g_{ij}'^*$, $\mathbf{a}'$, a_4', f_{ij}' are set to be $\boldsymbol{\varphi}$, $\boldsymbol{\varphi}^*$, g_{ij}, g_{ij}^*, $\mathbf{a}$, a_4, and f_{ij}, respectively.

Similar to (A71), we define

$$\begin{aligned} \varphi_4 &\equiv \varphi_4' - (\partial G/\partial\varphi_4'^*)(1/m^2), \\ \varphi_4^* &\equiv \varphi_4'^* - (\partial G/\partial\varphi_4')(1/m^2), \\ g_{4j} &\equiv g_{4j}' - (\partial G/\partial g_{4j}'^*), \\ g_{4j}^* &\equiv g_{4j}'^* - (\partial G/\partial g_{4j}'), \end{aligned}$$

and

$$f_{4j} \equiv f_{4j}' - (\partial G/\partial f_{4j}'). \tag{A76}$$

It is straightforward (though somewhat tedious) to show that the function H_{int} (A29) is related to G by

$$\begin{aligned} H_{\text{int}} &= \frac{1}{m^2}\left(\frac{\partial G}{\partial\varphi_4'}\right)\left(\frac{\partial G}{\partial\varphi_4'^*}\right) \\ &\quad + \left(\frac{\partial G}{\partial g_{4j}'}\right)\left(\frac{\partial G}{\partial g_{4j}'^*}\right) + \frac{1}{2}\left(\frac{\partial G}{\partial f_{4j}'}\right)\left(\frac{\partial G}{\partial f_{4j}'}\right) - G, \end{aligned} \tag{A77}$$

where the 11 primed field variables are regarded as functions of the unprimed variables by using (A76).

In order to use Lemma 2, we define 11 Hermitian variables $\psi_1, \cdots \psi_{11}$ by

$$\begin{aligned} \psi_j' &= if_{4j}', \\ \psi_{3+j}' &= (1/\sqrt{2})[g_{4j}' - g_{4j}'^*], \\ \psi_{6+j}' &= -i(1/\sqrt{2})[g_{4j}' + g_{4j}'^*], \\ \psi_{10}' &= (m/\sqrt{2})[\varphi_4' - \varphi_4'^*], \end{aligned}$$

and

$$\psi_{11}' = -i(m/\sqrt{2})[\varphi_4' + \varphi_4'^*], \tag{A78}$$

where $j=1, 2, 3$. Regarding G as a function of ψ_i', we find that equalities identical with (A78) hold between $\psi_1, \cdots \psi_{11}$ [which are defined by (A71)] and f_{4j}, g_{4j}, g_{4j}^*, φ_4 and φ_4^* [which are defined by (A76)]. Therefore, (A77) implies the validity of (A72). Furthermore, we notice that comparison between (A39) and (A34) shows that (A69) is satisfied.

Theorem 3, thus, becomes a special case of Lemma 2 provided one can show that the δH given by (A68) is, indeed, equal to (A38).

By using (A70) and (A74), the symmetric matrix A is found to be

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_1 & I_1 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_2 & I_2 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 & I_3 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(ea_1/m) \\ & & & & 0 & 0 & 0 & 0 & 0 & 0 & -(ea_2/m) \\ & & & & & 0 & 0 & 0 & 0 & 0 & -(ea_3/m) \\ & & & & & & 0 & 0 & 0 & (ea_1/m) & 0 \\ & & & & & & & 0 & 0 & (ea_2/m) & 0 \\ & & & & & & & & 0 & (ea_3/m) & 0 \\ & & & & & & & & & (e^2\mathbf{a}\cdot\mathbf{a}/m^2) & 0 \\ & & & & & & & & & & (e^2\mathbf{a}\cdot\mathbf{a}/m^2) \end{bmatrix}, \tag{A79}$$

where

$$R_j = (e\kappa/m)(1/\sqrt{2})(\varphi_j^* + \varphi_j),$$

and

$$I_j = i(e\kappa/m)(1/\sqrt{2})(\varphi_j^* - \varphi_j) \quad (j=1, 2, 3).$$

Utilizing the identity,

$$\text{trace}[\ln(1+A)] = \ln(\det|1+A|),$$

one finds that (A38) is true. Theorem 3 is, therefore, proved.

APPENDIX D. DERIVATIONS OF FEYNMAN RULES IN ξ-LIMITING FORMALISM

In the ξ-limiting formalism the interaction Lagrangian contains only a single time derivative of the electromagnetic field. Therefore, the results given in Figs. 2 and 3 can be directly obtained by using Lemma 1 and setting the matrix $A=0$.

APPENDIX E. REMARKS ON THE ORIGIN OF $\delta^4(0)$ TERM IN FIGURE 1

It is clear by comparing Figs. 1 and 2 (or Figs. 1 and 3) that for a given process, to the lowest order in e, the

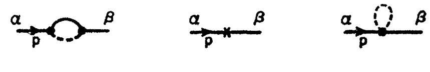


FIG. 6. Feynman diagrams for self-energy of mesons (discussed in Appendix E).

Feynman diagram does not contain any closed loops and therefore has the same value in the ξ-limiting process when $\xi \to 0$ as in the usual canonical formalism.

For higher order Feynman diagrams, the additional vertices of Fig. 1 carrying factors $\delta^4(0)$ which are infinite must be included. They are explicitly noncovariant under Lorentz transformations. The origin of these noncovariant terms is the nonidentical treatment of the space and time components of the meson field in the usual canonical formalism, as we now illustrate in the following example.

Consider the self-energy of a meson to the order e^2. There are three Feynman diagrams that contribute, as illustrated in Fig. 6 where the cross in (b) stands for the additional vertices of Fig. 1. The most divergent terms come from (a) and (b). They are, respectively, per unit volume,

$$A_a = \frac{i\kappa^2 e^2}{(2\pi)^4 m^2} \varphi_\beta^* \varphi_\alpha \int \frac{k^2 \delta_{\alpha\beta} - k_\alpha k_\beta}{(p+k)^2 + m^2} d^3k dk_0, \quad \text{(A80)}$$

and

$$A_b = -\frac{i\kappa^2 e^2 \delta^4(0)}{m^2} \boldsymbol{\varphi}^* \cdot \boldsymbol{\varphi}. \quad \text{(A81)}$$

We can make the same calculation using the ξ-limiting formalism. Only diagrams (a) and (c) contribute, and the most divergent term comes from (a): This most divergent term can be written as

$$B_a = A_a - \frac{i\kappa^2 e^2}{(2\pi)^4 m^2} \varphi_\beta^* \varphi_\alpha \int \Delta d^3k dk_0, \quad \text{(A82)}$$

where

$$\Delta = \frac{k^2 \delta_{\alpha\beta} - k_\alpha k_\beta}{(p+k)^2 + m^2} \frac{k^2 + m^2}{k^2 + \xi^{-1} m^2}, \quad \text{(A83)}$$

which is covariant.

Let us now evaluate the integral in (A82) by first integrating over k_0, then making $\xi \to 0$. Now,

$$\Delta \sim -\mathbf{k}^2/k_0^2 \quad \text{as} \quad k_0 \to \infty \quad \text{for} \quad \alpha = \beta = 4$$

and

$$\Delta \sim 1 \quad \text{as} \quad k_0 \to \infty \quad \text{for} \quad \alpha = \beta = 1, 2, 3.$$

Thus, as $\xi \to 0$

$$\int \Delta dk_0 \sim \int dk_0 \quad \text{for} \quad \alpha = \beta = 1, 2, 3$$

$$\sim 0 \quad \text{otherwise.}$$

It is clear that if we evaluate the integral in (A82) by first integrating over k_0, then taking $\xi \to 0$, then integrating over $\mathbf{k}$, we obtain

$$\text{(A82)} = \text{(A80)} + \text{(A81)}.$$

This example illustrates the fact that the ξ-limiting formalism is an explicitly covariant method which is more convenient than the canonical formalism.